Readdressing the hierarchy problem in a Randall-Sundrum scenario with bulk Kalb-Ramond background

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We re-examine the fine tuning problem of the Higgs mass, when an antisymmetric two form Kalb-Ramond (KR) field is present in the bulk of a Randall-Sundrum (RS) braneworld. Taking into account the back-reaction of the KR field, we obtain the exact correction to the RS metric. The modified metric also warps the Higgs mass from Planck scale (in higher dimension) to TeV scale (on the visible brane) for a range of values of kr exceeding the original RS value (where k = Planck mass and r = size of extra dimension). However, it requires an extraordinary suppression of the KR field density, indicating the re-appearance of the fine tuning problem in a different guise. The new spacetime also generates a small negative cosmological constant on the visible brane. These results are particularly relevant for certain string based models, where the KR field is unavoidably present in the bulk. We further show that such a bulk antisymmetric KR field fails to stabilize the braneworld.

Despite the success of the standard model of elementary particles in explaining physical phenomena up to the electroweak scale, the radiative instability of the Higgs mass remains an unresolved issue. The minimal supersymmetric standard model (MSSM) can solve this problem, but not without the introduction of a host of superpartners in the theory. As viable alternatives to supersymmetry, two models have recently been proposed, both of which require the presence of one or more extra spatial dimension(s). These models are respectively known as the ADD (named after Arkani-Hamed, Dimopoulos and Dvali) [1] and RS (named after Randall and Sundrum) [2]. For other non-supersymmetric compactifications which reproduce the standard model at low energies, see e.g. [4].

In particular, in the RS approach, one considers a 5 dimensional anti de-sitter spacetime with the extra spatial dimension orbifolded as S_1/Z_2. Two $(3 + 1)$- dimensional branes, known as the visible (TeV) brane and hidden (Planck) brane are placed at the two orbifold fixed points with the following bulk metric ansatz:

$$ds^2 = \exp{(-A)}\eta_{\mu\nu}dx^\mu dx^\nu - r^2 d\phi^2 \ , \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$ (i.e. the visible coordinates) and ϕ is the hidden coordinate ($\eta_{\mu\nu}$ is the usual 4-dimensional Minkowski metric, whereas G_{MN} etc will denote the full five-dimensional metric). As a consequence of the warped geometry, all mass scales get exponentially lowered from the Planck scale on the hidden brane to the TeV scale on the visible brane. The radius of the compact dimension, on the other hand, remains close to the Planck length, the 4 and 5 dimensional Planck scales remain close to each other and no scale hierarchy problem appears in this case. In both the above models, the standard model fields are assumed to be localized on the visible brane, whereas gravity alone propagates in the bulk. This is consistent with string theory, where the standard model fields appear as open string modes attached to the visible brane, whereas gravity being a closed string excitation, propagates in the bulk. Attempts to stabilize such a braneworld include those which incorporate a scalar field in the bulk, resulting in an effective potential for the compactification radius for the RS model [5]. Such a model however does not take into account the back-reaction of the scalar field on the background metric. Several other pieces of work followed, which compute the scalar back-reaction on the metric [6]. But except for some special cases, an exact back-reacted solution has not been derived.

Our present work is motivated from a different angle. In the context of string theory, apart from the graviton and scalar excitations, the second rank anti-symmetric tensor field (the KR field) in the Neveu-Schwarz-Neveu-Schwarz

(NS-NS) sector of the underlying string theory, can also be present in the bulk as a closed string mode. Various aspects of the presence of the antisymmetric fields have been discussed in the context of D-brane models [7]. Moreover ref.[8] explored various cosmological implications of the KR field in RS background. In this paper we explore the effect of the KR field on the background metric and re-examine the fine-tuning problem. We first derive an exact solution for the metric and exhibit its deviation from the RS solution. The new metric depends on the energy density of the KR field, and goes to the RS solution smoothly in the limit of KR energy density tending to zero. This scenario solves the hierarchy problem as well, not just for the value of kr predicted by RS, but for any value greater than the RS value. However, it requires the KR energy density to be exceedingly small, signalling the return of the fine tuning problem in a different guise. This value nonetheless matches remarkably well with the KR energy density on the visible brane, calculated from the solution of the KR field in a Randall Sundrum braneworld in a previous work [9]. Moreover, the KR density induces a negative cosmological constant in the visible brane.

We begin with the RS metric ansatz (1), and the action

$$S = S_{Gravity} + S_{vis} + S_{hid} + S_{KR} \,, \tag{2}$$

$$\text{where,} \quad S_{Gravity} = \int d^4x \, r \, d\phi \sqrt{G} \, [2M^3 R + \Lambda] \tag{3}$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}} \, [L_{vis} - V_{vis}] \tag{4}$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}} \, [L_{hid} - V_{hid}] \tag{5}$$

$$S_{KR} = \int d^4x \, r \, d\phi \sqrt{G} \, [2H_{MNL}H^{MNL}] \,. \tag{6}$$

Here Λ is the five dimensional cosmological constant, V_{vis}, V_{hid} are the visible and hidden brane tensions. $H_{MNL} = \partial_{[M}B_{NL]}$ is the third rank antisymmetric field strength corresponding to the two-form KR field B_{MN} [9]. The 5 dimensional Einstein equations are as follows (where $' \equiv d/d\phi$):

$$\frac{3}{2}A'^2 = -\frac{\Lambda}{4M^3} \, r^2 - \frac{3}{2M^3}g^{\nu\beta}g^{\lambda\gamma}H_{\phi\nu\lambda}H_{\phi\beta\gamma} \tag{7}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3}r^2 + \frac{\exp(-2A)}{2M^3} \, \eta^{\lambda\gamma}[-12\eta^{00}H_{\phi0\lambda}H_{\phi0\gamma} + 3\eta^{\nu\beta}H_{\phi\nu\lambda}H_{\phi\beta\gamma}] \tag{8}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3}r^2 + \frac{\exp(-2A)}{2M^3} \, \eta^{\lambda\gamma}[-12\eta^{ii}H_{\phi i\lambda}H_{\phi i\gamma} + 3\eta^{\nu\beta}H_{\phi\nu\lambda}H_{\phi\beta\gamma}] \tag{9}$$

In Eq.(9), the index i on the right hand side runs over 1,2 and 3, i.e. three spatial components x, y, z, and there is no sum over i. Also, $\eta^{ij} \equiv g^{im}g^{jn}\eta_{mn}$. Adding Eq.(8) and the x, y, z components of Eq.(9), we get,

$$A'^2 - A'' = -\frac{\Lambda}{6M^3}r^2 \tag{10}$$

which has the solution ($k \equiv \sqrt{-\Lambda/24M^3}$; $b, c =$ integration constants):

$$\exp(-A) = \frac{\sqrt{b}}{2kr} \cosh(2kr\phi + 2krc) \,, \tag{11}$$

Although $H_{\mu\nu\lambda}$ terms seem to have disappeared from Eq.(10), it follows from Eq.(7) that the parameter b is proportional to the energy density of the KR field. The constant c needs to be determined from the boundary conditions. Imposing the condition $A(\phi = 0) = 0$ in Eq.(11), we get:

$$\frac{2kr}{\sqrt{b}} = \cosh(2krc) \,. \tag{12}$$

In the limit $c \to -\infty$, Eq.(11) becomes:

$$\exp(-A) = \frac{\sqrt{b}}{4kr} \exp(-2krc) \exp(-2kr\phi) \,,$$

while Eq.(12) becomes:

$$\frac{4kr}{\sqrt{b}} = \exp(-2krc) \,.$$

(This shows that $b \to 0$, i.e. the KR field vanishes in the above limit). Substituting the latter in the former, we get:

$$\exp(-A) = \exp(-2kr\phi) \ ,$$

which is the Randall-Sundrum result for the warp factor. Further, using Eq.(11) and the following condition near $\phi = 0, \pi$:

$$A'' \ = \ \frac{r}{6M^3} \left[\ V_{hid} \ \delta(\phi) + V_{vis} \ \delta(\phi - \pi) \ \right] \ , \tag{13}$$

$$\text{we get,} \quad c \ = \ -\frac{1}{2kr} \tanh^{-1}\left(\frac{V_{hid}}{24M^3k} \right) = -\pi + \frac{1}{2kr} \tanh^{-1}\left(\frac{V_{vis}}{24M^3k} \right) \ , \tag{14}$$

or equivalently:

$$V_{vis} = 24M^3k \tanh\left(2kr(\pi + c)\right) = -\frac{\Lambda}{k} \tanh\left(2kr(\pi + c)\right) \ , \quad V_{hid} = -24M^3k \tanh\left(2krc\right) = \frac{\Lambda}{k} \tanh\left(2krc\right) \ . \tag{15}$$

Once again, note that in the $c \to -\infty$ limit, one gets $V_{hid} = -V_{vis} = 24M^3k$, as in [2]. Once the metric is determined, we examine the condition under which the desired reduction of the Higgs mass from the Planck scale to the TeV scale can be achieved. From the warp factor given by Eq.(11) and Eq.(12), it follows that the Higgs mass at the visible brane is:

$$m_H^2 = \frac{\sqrt{b}}{2kr} \cosh\left[2kr\pi + \cosh^{-1} \frac{2kr}{\sqrt{b}} \right] m_0^2 = \left[\cosh(2kr\pi) - \sinh(2kr\pi) \sqrt{1 - \frac{b}{(2kr)^2}} \right] m_0^2 \ , \tag{16}$$

where $m_H \approx$ TeV and $m_0 \approx 10^{16}$ TeV denotes the mass parameter on the Planck (hidden) brane, such that $m_H/m_0 = 10^{-16}$. As expected, in the limit $b \to 0$, the relation $m_H = \exp(-kr\pi) \ m_0$ is recovered, from which one obtains $kr = (16/\pi) \ln(10) = 11.7269\ldots$. We will call this the RS value of kr. Eq.(16) can be inverted to obtain:

$$b = (2kr)^2 \left[1 - \left(\coth(2kr\pi) - (m_H/m_0)^2 \text{cosech}(2kr\pi) \right)^2 \right] \ . \tag{17}$$

It follows from the last equation that $b = 0$ at the RS value of kr, $b < 0$ for $kr <$ the RS value and $b > 0$ for $kr >$ the RS value. For $kr \gg$ RS value, $b \to 0$, whereas for $kr \to 0$, the asymptotic value of $b = -1/\pi^2 = -0.1013\ldots$ is reached. Since positivity of b is required for the metric to be real (see Eq.(11)), we conclude that for a non-vanishing KR field, however small, kr has to exceed the RS value. The plot of $\log|b|$ vs. kr is shown in Fig.1. Note that there is no upper bound on kr, and in principle it can be as large as desired. In other words, the hierarchy problem can be solved for any value of $kr >$ the RS value, the corresponding value of the KR field given by Eq.(17). The kink in the graph corresponds to the RS value, when $b = 0$ and it can be seen that as kr increases from the RS value, b reaches a maximum of about 10^{-61} and then falls rapidly to zero. In other words, the KR field in the observable universe has to be exceedingly small! It is interesting to note that $b \approx 10^{-62}$ was also predicted in [9]. There, the KR field in the bulk was decomposed into various Kaluza-Klein (KK) modes and the equations of motion for these modes were obtained. The solutions and the masses for each of these KK modes then followed from these equations, and one obtained $H_{\mu\nu\lambda} \approx M^{3/2}10^{-31}$, from which it followed that $b \approx 10^{-62}$, similar to what we obtained here!

Finally, we compute the effective four dimensional cosmological constant on the visible brane [10]. From (15), we get:

$$\lambda \equiv \frac{1}{2} \left(kV_{vis} + \Lambda \right) \ = \ 12M^3k \left[\tanh(2krc) + 1 \right] \tag{18}$$

$$\approx \ -24M^3k \ \exp(4krc) \tag{19}$$

$$= \ -24M^3k \ \frac{b}{(4kr)^2} \tag{20}$$

$$= \ -6M^3k \left[1 - \left(\coth(2kr\pi) - (m_H/m_0)^2 \text{cosech}(2kr\pi) \right)^2 \right] \ . \tag{21}$$

where the limit $c \to -\infty$ has been taken to obtain Eq.(19), Eq.(12) has been used in the same limit to obtain Eq.(20) and Eq.(17) has been used to obtain (21). Note that the cosmological constant gets determined in terms on kr alone. However, for any non-zero $b > 0$, or $kr >$ RS value, its signature is negative and it attains a minimum value of about -10^{-63} (in Planck units). Thus, if we take the accepted value of $\lambda \approx 10^{-123}$ (in Planck units) seriously, it appears that one would have to invoke other fields to cancel the negative contribution to λ from the KR field.
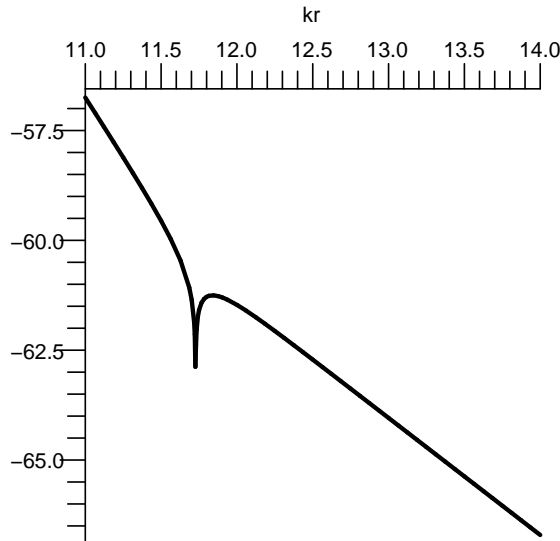
FIG. 1: Plot of $\log|b|$ vs. kr for $kr = 11\ldots14$. The kink corresponds to $kr = 11.7269\ldots$, the RS value, where $b = 0$ and $\log|b| \to -\infty$. Above the RS value, b rises to a maximum of 10^{-62} and then decays to zero. Below the RS value, $b < 0$, and the corresponding metric is unphysical. The curve meets the $\log|b|$ axis at $\log(1/\pi^2) = -0.9943$ (not shown).

Finally, to explore whether the KR field helps to stabilize the braneworld, we follow the prescription of [5]. Remarkably we do not need the solution for the KR field to obtain the induced potential that may help in stabilization. We take Eq.(7) along with the solution for the metric, plug it back in the KR action, and integrate over the compact coordinate ϕ, to arrive at the following effective potential,

$$V = -12\pi\frac{M^3 b}{r} \ . \tag{22}$$

Clearly, this potential does not have a minimum for $r \neq 0$, indicating that stabilization of the braneworld by KR fields may not be possible.

We conclude with the following observations. Presence of a bulk KR field yields a new warped solution for the metric which is also capable of suppressing the Higgs mass on the visible brane, provided (i) kr exceeds the value originally predicted by RS but otherwise can remain unbounded, and (ii) the parameter b, representing the KR field, is heavily suppressed. We interpret the second requirement as the effective re-appearance of the fine tuning problem. Furthermore, the KR field generates an effective negative cosmological constant on the visible brane. Stabilization of the braneworld however, cannot be achieved even if the KR field is considered in the bulk along with gravity. We hope to further report on these and related issues elsewhere [11].

Acknowledgement

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